

D. Comelli⁽¹⁾, M. Pietroni⁽²⁾ and A. Riotto⁽²⁾⁽¹⁾*INFN - Sezione di Ferrara, via Paradiso 12, I-35131 Ferrara, Italy*⁽²⁾*INFN, Sezione di Padova, via Marzolo 8, I-35131, Padova, Italy*

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It is a puzzle why the densities of dark matter and dark energy are nearly equal today when they scale so differently during the expansion of the universe. This conundrum may be solved if there is a coupling between the two dark sectors. In this paper we assume that dark matter is made of cold relics with masses depending exponentially on the scalar field associated to dark energy. Since the dynamics of the system is dominated by an attractor solution, the dark matter particle mass is forced to change with time as to ensure that the ratio between the energy densities of dark matter and dark energy become a constant at late times and one readily realizes that the present-day dark matter abundance is not very sensitive to its value when dark matter particles decouple from the thermal bath. We show that the dependence of the present abundance of cold dark matter on the parameters of the model differs drastically from the familiar results where no connection between dark energy and dark matter is present. In particular, we analyze the case in which the cold dark matter particle is the lightest supersymmetric particle.

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1. Introduction. Combined analysis of cosmological observations such as cosmic microwave radiation (CMB) anisotropies [1], cluster baryon fraction [2] and supernovae Ia [3], give increasing support for the so called ‘cosmic concordance’ model, in which the universe is flat ($\Omega_{\text{tot}} = 1$), and made for one third of non-relativistic dark matter (DM), and for two thirds of a smooth component, called Dark Energy (DE). DM is commonly associated to weakly interacting particles (WIMPs), and can be described as a fluid with vanishing pressure. DE, being responsible for the accelerated expansion of the universe, has to be identified with some more exotic component with negative pressure, such as the cosmological constant or a scalar field with a proper potential [4]. A common assumption is that there is no interaction between DM and DE. This implies that the ratio between their energy densities evolves in time as $\rho_{\text{DM}}/\rho_{\text{DE}} \sim a^{3W}$, a being the scale factor of the universe and W the DE equation of state ($W = -1$ for a cosmological constant, $W < -1/3$ to get acceleration). As a consequence, the cosmological constant or the scale of the effective potential have to be carefully tuned in order to get the observed proportions of DM and DE right today.

A possible solution to this ‘cosmic coincidence’ problem may be obtained by introducing a coupling between DM and DE. More specifically, if DE is identified with a scalar field φ with an exponential potential and the DM particle mass also depends exponentially on φ , the late time behavior of the cosmological equations gives accelerated expansion and, at the same time, a constant DM/DE ratio [5]. This behavior relies on the existence of an *attractor* solution, which makes the late time cosmology insensitive to the initial conditions for DE and DM. The possibility of varying mass particles (VAMPs) as DM candidates was considered previously in [6], but

with different functional dependences not leading to a solution of the cosmic coincidence problem.

In this letter, we will explore the impact of this scenario on the usual cold DM picture, which is based on the freeze-out at an early epoch of some non-relativistic species (see, for instance, [7]). In the standard scenario with constant mass particles the late time DM energy density is just the one at the freeze-out rescaled by the volume expansion. Instead, in the present VAMP scenario the attractor prompts a mass change in order to have the right DM/DE ratio today. As we will see, besides solving the cosmic coincidence problem, this will significantly enlarge the parameter space allowed by the observed abundances.

2. VAMPs and the coincidence problem We will consider as DM candidate a particle χ of mass M , depending exponentially on the DE field φ ,

$$M_\chi(\varphi) = \overline{M} e^{-\lambda\varphi}, \quad (1)$$

where φ is expressed in units of the Planck Mass M_p . The scalar field has an exponential potential

$$V(\varphi) = \overline{V} e^{\beta\varphi}, \quad (2)$$

with $\lambda, \beta > 0$. If the DM particle is non-relativistic, its energy density is also φ -dependent, since it is given by

$$\rho_\chi = M_\chi(\varphi) Y_\chi n_\gamma, \quad (3)$$

where $Y_\chi = n_\chi/n_\gamma$ is the number density of DM particles relative to the density of photons, $n_\gamma = n_\gamma^0 (a/a_0)^{-3}$, with $n_\gamma^0 = 411 \text{ cm}^{-3}$, and a/a_0 the scale factor of the universe relative to its value today.

As a consequence, the scalar field is subject to an effective potential given by the sum of Eqs. (2) and (3). The equation of motion is then

$$\frac{1}{3} \frac{\rho_\chi + \rho_b + V + \rho_{\text{rad}}}{1 - \varphi'^2/6} \varphi'' + \frac{1}{2} \left(\rho_\chi + \rho_b + 2V + \frac{2}{3} \rho_{\text{rad}} \right) \varphi' = -\beta V + \lambda \rho_\chi, \quad (4)$$

where primes denote derivatives with respect to $\xi = \log(a/a_0)$, ρ_b and ρ_{rad} are the energy densities in baryons and radiation, respectively, and we have assumed a spatially flat universe. Since we are interested in the late-time behavior, we can assume $\rho_b, \rho_{\text{rad}} \ll \rho_\chi, \rho_\varphi$. In this limit it is easy to see that there is a solution such that

$$\varphi = \frac{-3}{\lambda + \beta} \xi, \quad \Omega_\varphi \simeq 1 - \Omega_\chi = \frac{3 + \lambda(\lambda + \beta)}{(\lambda + \beta)^2}, \quad (5)$$

which is an *attractor* in field space if $\beta > (-\lambda + \sqrt{\lambda^2 + 12})/2$. From the moment the attractor is reached, the energy densities in DM and in DE evolve at a constant ratio depending only on λ and β , thus solving the cosmic coincidence problem.

A closer inspection at the solution (5) using Friedman's equations reveals that

$$\frac{a \ddot{a}}{\dot{a}^2} = -\frac{1 + 3W}{2}, \quad \text{with} \quad W = \frac{-\lambda}{\beta + \lambda}, \quad (6)$$

that is W is negative and may lead, if $W < -1/3$, to an accelerated expansion of the universe. To understand how it is possible to get both acceleration and constant ratio between DM and DE one may look at the scaling behavior of the energy densities on the attractor (5),

$$\rho_\chi \sim e^{-\lambda\varphi - 3\xi} \sim \rho_\varphi \sim e^{\beta\varphi} \sim e^{-3(W+1)\xi}. \quad (7)$$

The φ dependence of the DM mass modifies the usual scaling $e^{-3\xi}$ of non relativistic matter. Since the mass increases with the expansion, it corresponds to an effectively negative W , even though DM is still a pressureless fluid made up by non-interacting particles.

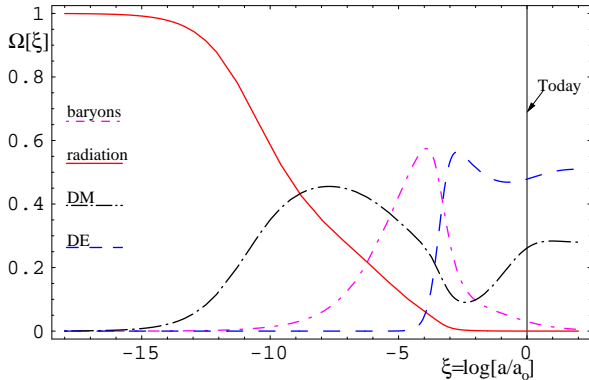


FIG. 1. Time evolution of the relative abundance of different species, expressed as fractions of the critical density. The corresponding parameter configuration is: $\beta = 1.88$, $\lambda = 0.82$, $\bar{V} = 10^2 \text{ GeV}^4$, $M_S = 500 \text{ GeV}$, $M_\chi^{\text{ph}} = 105 \text{ GeV}$ and $\alpha_S^2 = 0.0068$

In Fig. 1 we plot a typical solution, including also ρ_b and ρ_{rad} . During radiation domination $\rho_\varphi, \rho_\chi \ll \rho_{\text{rad}}$, then $\varphi \sim \text{const.}$, so that DM exhibits the standard scaling, $e^{-3\xi}$. Around matter-radiation equivalence the attractor starts to become effective and after some transient regime – possibly including a baryon-dominated epoch – it is reached at about the present epoch. In this scenario, the future will be characterized by values of Ω_φ , and Ω_χ closer and closer to the values in (5), with ρ_b and ρ_{rad} diluting with the usual laws, a^{-3} and a^{-4} , respectively.

As will be discussed in [8] the growth of fluctuations in this scenario is compatible with the present data on CMB and large scale structure. In particular, it is remarkable that matter fluctuations grow even during accelerated expansion, unlike in the usual Λ -dominated or quintessence scenario.

2. VAMPs and DM abundance As we have discussed in the previous section, the late-time attractor forces the DM particle mass to change with time as to ensure that DM and DE scale at the same, constant, ratio given by Eq. (5). In this section we show how this translates into an enhancement of the parameter space giving the observed energy densities. Alternative ideas to modify the standard freeze-out picture of CDM suppose a kinton-dominated expansion of the universe [9] or a low-reheating temperature after inflation [10]. Differently from these approaches, where some new dynamics occurs at early epochs, in our proposal the standard expansion history of the universe gets modified only at late times – after radiation-matter equality – when the field φ starts to roll. We assume that the DM particle χ falls into the category of Cold Dark Matter (CDM), that is χ -particles cease annihilating during the evolution of the universe when they were non-relativistic (in the case of relativistic DM, due to the fact that the DM abundance Y_χ is mass independent, the predictions of the VAMP and the standard scenarios are the same). This amounts to assuming that $x_f = M_\chi(\varphi_f)/T_f > 1$, where φ_f is the value of the DE field at the freeze-out epoch and T_f is the freeze-out temperature. The present DM energy density, Eq. (3), depends on the value of the *physical* χ -mass today $M_\chi^{\text{ph}} \equiv M_\chi(\varphi_0) \equiv \bar{M}e^{-\lambda\varphi_0}$ (where φ_0 is the value of the DE field today) and on the comoving number density Y_χ . The latter depends, among other parameters, on the χ mass at the earlier epoch of freeze-out, *i.e.* on $M_\chi(\varphi_f) \equiv \bar{M}e^{-\lambda\varphi_f} = M_\chi^{\text{ph}}e^{\lambda\Delta\varphi}$, where $\Delta\varphi \equiv \varphi_0 - \varphi_f$.

In order to be definite, we will consider the case of stable χ -particles annihilating into light states by exchanging an heavier particle S of mass M_S , $\chi\bar{\chi} \rightarrow \text{light states}$. To estimate x_f we make use of the criterion $\Gamma_{\text{ann}}(x_f) \simeq H(x_f)$, where $\Gamma_{\text{ann}} = n_\chi^{\text{eq}} \langle \sigma_{\text{ann}} | v | \rangle$ and $\langle \sigma_{\text{ann}} | v | \rangle$ is the thermally-averaged annihilation cross-section times velocity of the DM particles. We also suppose that the DM particles annihilate through a p -wave suppression channel. This happens – for instance – if χ is a Majorana fermion.

Under these assumptions, the thermally-averaged annihilation cross section at freeze-out is given by

$$\langle\sigma_{\text{ann}}|v|\rangle = \alpha_S^2 \frac{T_f}{M_\chi(\varphi_f)} \frac{M_\chi^2(\varphi_f)}{(M_\chi^2(\varphi_f) + M_S^2)^2} \equiv \sigma_0 \frac{T_f}{M_\chi(\varphi_f)}, \quad (8)$$

where α_S is a combination of coupling constants measuring the strengths of the interactions. The standard freeze-out picture [7] gives the number abundance at late times $Y_\chi(T_0) \simeq Y_\chi(T_f) = (7.6x_f^2/g_*^{1/2}(T_f) M_p \sigma_0 M_\chi(\varphi_f))$, where $T_0 \simeq 2.75$ K is the present-day photon temperature and g_* counts the relativistic degrees of freedom in the plasma at the freeze-out epoch. The energy density of the DM particles today reads

$$\Omega_\chi(t_0)h^2 = c e^{-3\lambda\Delta\varphi} \frac{(e^{2\lambda\Delta\varphi} (M_\chi^{\text{ph}})^2 + M_S^2)^2}{(M_\chi^{\text{ph}})^2} \quad (9)$$

with $c \simeq 1.46 \times 10^{-10} \alpha_S^{-2} (x_f^2/g_*^{1/2}(T_f)) \text{ GeV}^{-2}$. The standard ΛCDM scenario is recovered by setting $\Delta\varphi = 0$ or, equivalently, $\lambda = \beta = 0$. In this case, \bar{V} plays the role of the cosmological constant, $M_\chi = M_\chi^{\text{ph}}$ at any epoch, and constant $\Omega_\chi(t_0)h^2$ contours in the $(M_\chi - M_S)$ plane are those corresponding to $\Phi = 1$ in Fig. 2, where the region $M_S < M_\chi$ has been excluded. It is worth while stressing that, once the acceptable range for $\Omega_\chi(t_0)h^2$ has been chosen, the DE abundance has to be fine tuned to give a flat universe (at, say, ten percent accuracy). This translates into a constraint on \bar{V} which is nothing but the cosmic coincidence problem of the ΛCDM scenario.

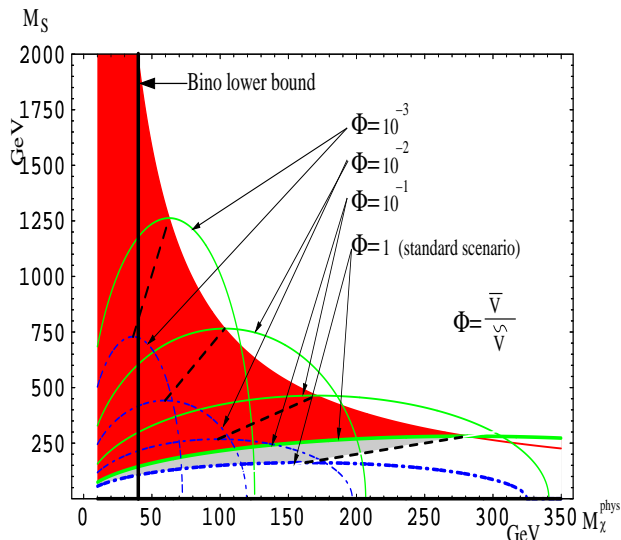


FIG. 2. Regions with $0.1 \leq \Omega_\chi h^2 \leq 0.3$ and $M_S > M_\chi$ for fixed values of $\Delta\varphi$, see Eq. (10). The grey region represents the parameter space available in the standard scenario $\Delta\varphi = 0$. The red+grey region represents the total parameter space available in the VAMP scenario.

In the VAMP scenario the situation changes dramatically. Keeping the physical abundance, $\Omega_\chi(t_0)h^2$, fixed and assuming the attractor has been reached by today, the circular contour for $\Delta\varphi = 0$ is deformed to an ellipse of the same area and eccentricity given by $e^{\lambda\Delta\varphi}$, as in Fig. 2. On the same contour the potential energy of the φ field is also constant, $\bar{V}e^{\beta\varphi_0}$, so we can express $\Delta\varphi$ in terms of \bar{V} as

$$\Delta\varphi = -\frac{1}{\beta} \log \frac{\bar{V}}{\tilde{V}} \equiv -\frac{1}{\beta} \log \Phi, \quad (10)$$

where \tilde{V} is the particular value of \bar{V} such that $\Delta\varphi = 0$.

Compared to the ΛCDM case, in the VAMP case we can vary \bar{V} by many orders of magnitude, see Fig. 2. In principle, any value of \bar{V} would give an acceptable ellipse. In practice, extreme values would prevent the field φ from reaching the attractor before the present epoch, thus deforming the corresponding contour from an elliptical shape and reducing its area. However, as we have checked numerically, the allowed values for \bar{V} typically span many orders of magnitude, thus removing any fine tuning on this parameter.

The most striking feature is that observationally allowed values of $\Omega_\chi(t_0)h^2$ can be obtained for very large masses of the scalar particle S . In the traditional ΛCDM scenario large scalar masses would lead to a suppression of the annihilation cross-section and therefore to an overabundance of DM particles today. On the other hand, in the case in which the χ -mass depends on the DE field, the initially large value of the χ -energy density at the freeze-out is adiabatically reduced not only by the expansion of the universe, but also by the fact that, due to the running of the DE field, the χ -mass is smaller today than at freeze-out. For consistency, we have to impose in this case that χ has always been lighter than S , which is ensured by the constraint $M_\chi(\varphi_f) < M_S$, giving the straight dashed lines in Fig. 2.

Even more remarkable from a phenomenological point of view is the drastic enhancement of the allowed parameter space in the $(M_\chi^{\text{ph}} - M_S)$ plane. Varying \bar{V} , the allowed regions span an area delimited by a hyperbole, which contains values for M_S much larger than in the standard scenario.

Points above the hyperbole give still the good DM/DE ratio, but fail in reproducing their absolute value, that is, the Hubble parameter today.

4. An illustrative example: supersymmetric dark matter. Of the many CDM candidates, the best motivated from a particle physics point of view is the supersymmetric neutralino if the latter is the lightest supersymmetric particle (LSP) [11]. Most of the supersymmetric models obtained from supergravity usually predict that the LSP is an almost pure B -ino. Therefore, to give an illustrative example of our previous findings, we consider the case in which the LSP is a B -ino and its mass M_B depends exponentially upon the time-dependent DE field

ϕ , $M_B = \bar{M}_B \exp(-\lambda\varphi)$.

Being the B -ino the supersymmetric partner of the $U(1)_Y$ gauge boson, it does not directly contribute at the one-loop level to the running of gauge couplings. This means that the time-dependence of the electromagnetic coupling is induced by the bino mass only through tiny quantities parametrizing how much the LSP composition depends on the Wino and Higgsinos. A full parameter space analysis regarding this specific point will be presented in [12]. In the early universe, B -inos mainly annihilate into fermion pairs through t -channel exchange of squarks and sleptons. Exceptions occur only for pathological situations in which there is a resonant s -channel exchange of Z^0 or a Higgs boson. Because of the large hypercharge of the right-handed electron and the expected lightness of sleptons compared to squarks, it is often a good approximation to include in the annihilation cross section only the exchange of the right-handed sleptons.

Summing over three slepton degenerate families with mass \tilde{M}_{ℓ_R} , the thermally-averaged B -ino annihilation cross section times velocity has the same form as eq. (8), with M_S and M_χ replaced by \tilde{M}_{ℓ_R} and M_B , respectively, and $\alpha_S^2 = 24\pi\alpha^2/\cos^4\theta_W \simeq 6.7 \times 10^{-3}$.

In the traditional case in which the mass of the B -ino is not dependent upon the DE field, $M_B = M_B^{\text{ph}}$, cosmological considerations give an upper bound to the B -ino mass. Indeed, the requirement that charged particles are not the LSP implies $\tilde{M}_{\ell_R} > M_B$. The minimum allowed B -ino relic abundance corresponds to the maximum annihilation cross section and therefore to the minimum \tilde{M}_{ℓ_R} . Setting $\tilde{M}_{\ell_R} = M_B$ in the expression for Ω , one obtains an upper bound on the B -ino mass of about 300 GeV (for $\Omega h^2 < 0.3$) [13]. This bound can be weakened in several ways. For example through the presence of resonant s -channel annihilations, once a small Higgsino admixture is introduced. Furthermore, as emphasized in Ref. [14], whenever the sleptons and the B -ino become degenerate in mass within about 10–20%, one cannot ignore the effects of coannihilation. These effects can modify significantly the B -ino relic abundance, because annihilation channels involving the charged sleptons have large cross sections which are not p -wave suppressed. Indeed, even in the case of the constrained model, the previous limit on the B -ino mass can be relaxed to about 600 GeV [14]. Coannihilation effects do not significantly modify the bound on the slepton mass for a fixed value of M_B (as long as it is not too close to \tilde{M}_{ℓ_R}).

On the other hand, these bounds on the slepton and B -ino masses can be drastically modified if the mass of the B -ino depends upon the DE field. These effects are illustrated in Fig. 2 at different values of the parameter \bar{V} . As expected from the findings of the previous section, observationally allowed values of $\Omega_B(t_0)h^2$ can be obtained for very large slepton masses for given values of \bar{V} : the smaller \bar{V} is, the larger the slepton mass can be. Furthermore, allowing \bar{V} to vary enhances drastically the allowed parameter space in the $(M_B^{\text{ph}} - \tilde{M}_{\ell_R})$ plane. We

leave a more complete analysis of the parameter space of the supersymmetric models for a future publication [12]. It is intriguing that a coupling between the dark energy and the dark matter sectors may provide not only a natural solution to the coincidence problem but also a drastic modification of the predicted abundance of CDM in terms of the particle physics parameters.

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